Structural Pricing of CoCos and Deposit Insurance with Regime Switching and Jumps

OLIVIER LE COURTOIS XIAOSHAN SU

AFFI Conference 2017

emlyon business School

Outline

References

- Diffusion structural models: Merton (JF, 1974), Black and Cox (JF, 1976), Longstaff and Schwartz (JF, 1995), Leland (JF, 1994a), Leland (WP, 1994b) and Leland and Toft (JF, 1996).
- Lévy structural models: Hilberink and Rogers (FS, 2002), Le Courtois and Quittard-Pinon (APFM, 2006) and Chen and Kou (MF, 2009).
- Regime switching EBIT-based model: Hainaut, Shen, and Zeng (AOR, 2016).

Jump and Regime Switching

The evolution of asset price includes important features such as jumps and regime switching.

- Short-term jump risk: In the short term, the evolution of asset price exhibits fairly extreme movements. The jumps phenomenon is produced by external shocks from some extreme events. From the statistical viewpoint, the asset return distribution shows skewness and high kurtosis features.
- Long-term regime switching risk: In the long term, structural changes in the macroeconomic conditions or in the business cycles cause modifications in the evolution pattern of the asset price.

Our Main Contributions

- We introduce a structural model that combines the jump and regime switching features using the regime switching double exponential jump diffusion process.
- We define a general Esscher transform that preserves the structure of the regime switching double exponential jump diffusion process where the interest rate, drift rate, volatility, but also jump intensity and jump distribution are regime switching.
- We use results of Jiang and Pistorius (2008) and provide numerical method to compute the matrix Wiener-Hopf factorization for the first passage time result of the regime switching double exponential jump diffusion process.
- We provide closed-form formulas for the value of the bank's equity, debt, deposits, CoCos and deposit insurance.
- We provide a empirical study where we study the influence of the regime switching risk and of jump risk on the conversion and default probabilities, on the value of balance sheet components and on the market required coupon rate of CoCos and fair premium rate of deposit insurance, etc.

The Regime Switching Jump Diffusion Structural Model

The value of the bank assets is assumed to follow an exponential regime switching jump diffusion process under the real-world probability measure P:

$$V_t = V_0 e^{X_t},$$

where V_0 is the initial bank asset value and X is a regime switching jump diffusion process:

$$X_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s + \int_0^t \mathrm{d}N_s.$$

- W is a standard Brownian motion.
- $J = \{J_t; t \geqslant 0\}$ is a continuous time Markov chain process on $(\Omega, \mathcal{F}, P)$ with a finite state space $E^0 = \{e_1, e_2, ..., e_n\}$ where $e_i = (0, ..., 1, ..., 0) \in \mathbb{R}^n$.
- $\mu_t = \langle \hat{\mu}, J_t \rangle$, $\sigma_t = \langle \hat{\sigma}, J_t \rangle$, $N_t = \langle \hat{N}, J_t \rangle$, and $\langle ., . \rangle$ denotes the inner product, $\hat{\mu} = (\hat{\mu}_1, \hat{\mu}_2, ..., \hat{\mu}_n)$, $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, ..., \hat{\sigma}_n)$ and $\hat{N} = (\hat{N}_1, \hat{N}_2, ..., \hat{N}_n)$.
- $\hat{\mu}_i \in \mathbb{R}$, $\hat{\sigma}_i \geqslant 0$, $\hat{N}_i = \{\hat{N}_i(t); t \geqslant 0\}$ is a compound Poisson process with rate $\hat{\lambda}_i$ and the jumps size is modeled with an asymmetric double exponential distribution of density function:

  $$f_i(y) = p_i \hat{\eta}_{1i} e^{-\hat{\eta}_{1i} y} I_{\{y \geqslant 0\}} + q_i \hat{\eta}_{2i} e^{\hat{\eta}_{2i} y} I_{\{y < 0\}},$$

  where $\hat{\eta}_{1i} > 1$, $\hat{\eta}_{2i} > 0$, $p_i \geqslant 0$, $q_i \geqslant 0$, $p_i + q_i = 1$.
- The stochastic processes $\{W_t; t \geqslant 0\}$ and $\{\hat{N}_i(t); t \geqslant 0\}$ are independent.

Esscher Transform Measure

We define the regime switching Esscher measure $\tilde{P}$ as follows:

$$S_t = \frac{d\tilde{P}}{dP}\Big|_{\mathcal{F}_t} = \frac{e^{\int_0^t \theta_s dX_s}}{E_P\left(e^{\int_0^t \theta_s dX_s} | \mathcal{G}_t\right)} = e^{\int_0^t \theta_s \sigma_s dW_s - \frac{1}{2}\int_0^t \theta_s^2 \sigma_s^2 ds} \frac{e^{\int_0^t \theta_s dN_s}}{E_P\left(e^{\int_0^t \theta_s dN_s} | \mathcal{G}_t\right)}$$

$$= e^{\int_0^t \theta_s \sigma_s dW_s - \frac{1}{2}\int_0^t \theta_s^2 \sigma_s^2 ds + \int_0^t \theta_s dN_s - \sum_{i=1}^{n} J_{it}\psi_i(\hat{\theta}_i)}.$$

and have the following lemma:

Lemma 1 S is an $(\mathcal{F}, P)$-martingale and the equivalent measure $\tilde{P}$ is well-defined.

- $\{\theta_t = \langle J_t, \hat{\theta}\rangle; t \geqslant 0\}$ is a non-negative $\mathcal{F}$—adapted stochastic process where $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_n)$.
- $\mathcal{G}$ is the filtration generated by J and $\mathcal{F} = \mathcal{G} \vee \mathcal{H}$ where $\mathcal{H}$ is the filtration generated by the $\{X^j; j = 1, 2, ..., n\}$ and X^j is X at state e_j.
- $J_{it} = \int_0^t \langle J_s, e_i\rangle$ is the occupation time of state e_i up to time t
- $\psi_j(u)$ is the Laplace exponent of $\hat{N}_1^j$:

$$\psi_j(u) = \ln E_P\left(e^{u\hat{N}_1^j}\right) = \hat{\lambda}_j\left(\frac{p_j\hat{\eta}_{1j}}{\hat{\eta}_{1j} - u} + \frac{q_j\hat{\eta}_{2j}}{\hat{\eta}_{2j} + u} - 1\right).$$

Esscher Transform Measure

We should choose the vector $(\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_n)$ to make the discounted asset price process a martingale. The martingale condition is deduced using the following lemma about the Laplace transform of the occupation times from Elliott and Osakwe (2006).

Lemma 2 For the $n-$state Markov switching model, the Laplace transform of the occupation times $Z = \{J_{1t}, J_{2t}, ..., J_{nt}\}$ is given by

$$\varphi(d) = E_P\left(e^{\langle d,Z\rangle}\right) = J_0{}'e^{\left(Q+\text{diag}(d)\right)t}\mathbf{1}, \qquad (1)$$

where $d = (d_1, d_2, ..., d_n)$, $\mathbf{1} \in \mathbb{R}^n$ is a vector of ones, J_0 is initial state of the Markov chain J and Q is the corresponding generator matrix.

The real solutions $\{\hat{\theta}_i, i = 1, 2, ..., n\}$ can be solved from the following martingale condition.

Proposition 1 The martingale condition is satisfied if and only if

$$\hat{\mu}_i - \hat{r}_i + \frac{1}{2}\hat{\sigma}_i^2 + \hat{\theta}_i\hat{\sigma}_i^2 + \psi_i(1+\hat{\theta}_i) - \psi_i(\hat{\theta}_i) = 0 \quad \forall i = 1, 2, ..., N. \qquad (2)$$

Esscher Transform Measure

Proposition 2 The process $\{X_t; t \geqslant 0\}$ keeps a regime switching double exponential jump diffusion structure under $\tilde{P}$. Let X be defined under $\tilde{P}$ as follows:

$$X_t = \int_0^t \mu_s^* \mathrm{d}s + \int_0^t \sigma_s^* \mathrm{d}W_s^* + \int_0^t \mathrm{d}N_s^* .$$

Then, W^* defined by $W_t^* = W_t - \int_0^t \theta_s \sigma_s \mathrm{d}s$ is a standard Brownian motion,

$\hat{\mu}_i^* = \hat{r}_i - \frac{1}{2}\hat{\sigma}_i^{*2} - \hat{\lambda}_i^* \left(\frac{p_i^* \hat{\eta}_{1i}^*}{\hat{\eta}_{1i}^* - 1} + \frac{(1-p_i^*)\hat{\eta}_{2i}^*}{\hat{\eta}_{2i}^* + 1} - 1 \right)$, $\hat{\sigma}_i^* = \hat{\sigma}_i$, $\hat{\lambda}_i^* = \hat{\lambda}_i \omega_i$, $p_i^* = \frac{1}{\omega_i} \left(\frac{p_i \hat{\eta}_{1i}}{\hat{\eta}_{1i} - \hat{\theta}_i} \right)$,
$\hat{\eta}_{1i}^* = \hat{\eta}_{1i} - \hat{\theta}_i$ and $\hat{\eta}_{2i}^* = \hat{\eta}_{2i} + \hat{\theta}_i$, where $\omega_i = E_P(e^{\hat{\theta}_i Y_i}) = \frac{p_i \hat{\eta}_{1i}}{\hat{\eta}_{1i} - \hat{\theta}_i} + \frac{(1-p_i)\hat{\eta}_{2i}}{\hat{\eta}_{2i} + \hat{\theta}_i}$.

The First Passage Time Problem

The first passage time problem across a constant level is related to the up-crossing and down-crossing ladder processes $\tilde{Y}^{+}$, $\tilde{Y}^{-}$ of the (A, Y).

- $A = \{A_t; t \geqslant 0\}$ is the fluid embedding of X and is a continuous process whose paths are constructed from the paths of X by replacing positive jumps by linear segments with slope $+1$ and negative jumps by linear segments with slope -1.
- $Y = \{Y_t; t \geqslant 0\}$ is an irreducible continuous time Markov chain process with a finite state space

$$E = E^{+} \cup E^{0} \cup E^{-}$$

where the spaces E^{0}, E^{+} and E^{-} correspond to the states where X moves as a pure diffusion, makes a positive jump and makes a negative jump, respectively.

The First Passage Time Problem

Specifically, A is represented as follows:

$$A_t = A_0 + \int_0^t u(Y_s)\mathrm{d}s + \int_0^t v(Y_s)\mathrm{d}W_s$$

where

$$u(j) = \begin{cases} 1 & \text{if } j \in E^+ \\ \hat{\mu}_j & \text{if } j \in E^0 \\ -1 & \text{if } j \in E^- \end{cases} \quad \text{and} \quad v(j) = \begin{cases} \hat{\sigma}_j & \text{if } j \in E^0 \\ 0 & \text{otherwise.} \end{cases}$$

The First Passage Time Problem

The generator matrix of Y is Q_0 where the $3n \times 3n$ matrix:

$$Q_{\hat{a}} = \begin{pmatrix} T^1 & t^1 & O_n \\ B^+ & Q - D_{\hat{a}} & B^- \\ O_n & t^2 & T^2 \end{pmatrix}.$$

We incorporate the regime switching discounting rate $\hat{a}$ into the process e^{wX_t} by changing the generator matrix of Y into $Q_{\hat{a}}$.

- The $n \times n$ matrix Q is the generator of J.
- $\hat{a} = (\hat{a}_1, \hat{a}_2, ..., \hat{a}_n) > 0$, $D_{\hat{a}} = \text{diag}(\lambda_i + \hat{a}_i)$, O_n is a zero matrix of size $n \times n$.
- $$B^+ = \begin{pmatrix} \hat{\lambda}_1 p_1 & & \\ & \ddots & \\ & & \hat{\lambda}_n p_n \end{pmatrix}, \quad B^- = \begin{pmatrix} \hat{\lambda}_1 (1 - p_1) & & \\ & \ddots & \\ & & \hat{\lambda}_n (1 - p_n) \end{pmatrix}$$

  and

  $$T^i = \begin{pmatrix} -\hat{\eta}_{i1} & & \\ & \ddots & \\ & & -\hat{\eta}_{in} \end{pmatrix}, \quad t^i = \begin{pmatrix} \hat{\eta}_{i1} & & \\ & \ddots & \\ & & \hat{\eta}_{in} \end{pmatrix}.$$

The First Passage Time Problem

The first passage time problem across a constant level is related to the up-crossing and down-crossing ladder process $\tilde{Y}^{+}$, $\tilde{Y}^{-}$ of the (A, Y). Denote as $Q^{(\hat{a},+)}$ and $Q^{(\hat{a},-)}$ the generator matrices of $\tilde{Y}^{+}$ and $\tilde{Y}^{-}$ and as the $n \times 2n$ matrices $\zeta^{(\hat{a},+)}$ and $\zeta^{(\hat{a},-)}$ the corresponding initial distributions.

The quadruple $(\zeta^{(\hat{a},+)}, Q^{(\hat{a},+)}, \zeta^{(\hat{a},-)}, Q^{(\hat{a},-)})$ is a Wiener-Hopf factorization of (A, Y) and the Wiener-Hopf factorization of (A, Y) is unique in the case that $\hat{a} > 0$.

- $\tilde{Y}^{+}$ and $\tilde{Y}^{-}$ are defined as follows:

  $$\tilde{Y}_z^{+} = Y(\tau_z^{+}) \quad \text{and} \quad \tilde{Y}_z^{-} = Y(\tau_z^{-})$$

  where

  $$\tau_z^{+} = \inf\{s \geqslant 0 : A_s > z\} \quad \text{and} \quad \tau_z^{-} = \inf\{s \geqslant 0 : A_s < z\}.$$

- The $\zeta^{(\hat{a},+)}$ and $\zeta^{(\hat{a},-)}$ are defined as follows:

  $$\zeta^{(\hat{a},+)}(i,j) = P_{0,i}(\tilde{Y}_0^{+} = j, \tau_0^{+} < \infty) \qquad \forall i \in E^{-}, j \in E^{+} \cup E^{0}$$

  and

  $$\zeta^{(\hat{a},-)}(i,j) = P_{0,i}(\tilde{Y}_0^{-} = j, \tau_0^{-} < \infty) \qquad \forall i \in E^{+}, j \in E^{-} \cup E^{0}.$$

The First Passage Time Problem

Definition 1 Denote as $(G^{(\hat{a},+)}, G^{(\hat{a},-)})$ the pair of irreducible $2n * 2n$ matrices, i.e. matrices with non-negative off-diagonal elements and non-positive row sums and $(\Pi^{(\hat{a},+)}, \Pi^{(\hat{a},-)})$ the pair of $n * 2n$ matrices with sub-probability vectors as rows. The quadruple

$$(\Pi^{(\hat{a},+)}, G^{(\hat{a},+)}, \Pi^{(\hat{a},-)}, G^{(\hat{a},-)})$$

is the Wiener-Hopf factorization of (A, Y) associated with $\hat{a} > 0$ if

$$\Xi(-G^{(\hat{a},+)}, W^{(\hat{a},+)}) = \Xi(G^{(\hat{a},-)}, W^{(\hat{a},-)}) = O,$$

where

$$\Xi(S, W) = \frac{1}{2}\Sigma^2 WS^2 + VWS + Q_{\hat{a}} W,$$

with the $3n \times 3n$ diagonal matrices:

$$\Sigma = \begin{pmatrix} O_n & & \\ & \begin{pmatrix} \hat{\sigma}_1 & & \\ & \ddots & \\ & & \hat{\sigma}_n \end{pmatrix} & \\ & & O_n \end{pmatrix}, \quad V = \begin{pmatrix} I_n & & \\ & \begin{pmatrix} \hat{\mu}_1 & & \\ & \ddots & \\ & & \hat{\mu}_n \end{pmatrix} & \\ & & -I_n \end{pmatrix},$$

and the $3n \times 2n$ matrices:

$$W^{(\hat{a},+)} = \begin{pmatrix} I_{2n} \\ \Pi^{(\hat{a},+)} \end{pmatrix} \quad W^{(\hat{a},-)} = \begin{pmatrix} \Pi^{(\hat{a},-)} \\ I_{2n} \end{pmatrix}$$

where I_n and I_{2n} are identity matrices of size $n \times n$ and of size $2n \times 2n$, respectively.

The First Passage Time Problem

The first passage time result of X has been given through the matrix Wiener-Hopf factorization of (A, Y) by Jiang and Pistorius (2008) and the following proposition is a consequence of their Theorem 3.

Proposition 3 Denote as τ the first passage time of X below a constant level b as

$$\tau = \inf\{t > 0 : X_t \leqslant b\}$$

and the contingent payoff $h(\tau) = \langle J_\tau, \hat{h} \rangle$ where $\hat{h} = (\hat{h}_1, ..., \hat{h}_n)$. Denote $a_t = \langle \hat{a}, J_t \rangle$, then

$$E\left(e^{-\int_0^\tau a_s \mathrm{d}s + wX_\tau} h(\tau) \right) = Y_0' W^{(\hat{a},-)} e^{Q^{(\hat{a},-)}(x-b)+wb} \tilde{h}$$

where w is a constant, x is the initial point of X, Y_0 is the initial state of Y,

$$\tilde{h} = \left(\left(\hat{h}_1, ..., \hat{h}_n \right), \left(\frac{\eta_{21}}{w+\eta_{21}} \hat{h}_1, ..., \frac{\eta_{2n}}{w+\eta_{2n}} \hat{h}_n \right) \right)',$$

and $Q^{(\hat{a},-)}$ is the Wiener-Hopf factor defined above.

Numerical Method

We now present a numerical method allowing us to compute $W^{(\hat{a},-)}$ and $Q^{(\hat{a},-)}$.

Step 1 We compute $2n$ different negative roots $\{\beta_i, i = 1, 2, ..., 2n\}$ from the equation $f(\beta) = \det(K(\beta)) = 0$ where

$$K(\beta) = \frac{1}{2}\Sigma^2\beta^2 + V\beta + Q_{\hat{a}}.$$

- Let ϑ be an eigenvector of $Q^{(\hat{a},-)}$ and let β be its associated eigenvalue. Right-multiply by ϑ the following equation:

$$\Xi(Q^{(\hat{a},-)}, W^{(\hat{a},-)}) = \frac{1}{2}\Sigma^2 W^{(\hat{a},-)} Q^{(\hat{a},-)2} + VW^{(\hat{a},-)} Q^{(\hat{a},-)} + Q_{\hat{a}} W^{(\hat{a},-)} = 0,$$

  and obtain:

$$\left(\frac{1}{2}\Sigma^2\beta^2 + V\beta + Q_{\hat{a}}\right) W^{(\hat{a},-)}\vartheta = 0.$$

- Denote $K(\beta) = \frac{1}{2}\Sigma^2\beta^2 + V\beta + Q_{\hat{a}}$. The matrix $K(\beta)$ is singular and $f(\beta) = \det(K(\beta)) = 0$. From Rogers and Shi (1994) and Rogers (1994), we have the following lemma:

  Lemma 3 Suppose that $\hat{a} > 0$. The equation $f(\beta) = 0$ has $4n$ different roots $\{\beta_i, i = 1, 2, ..., 4n\}$ and the roots are ranked as follows:

$$Re(\beta_1) \leqslant Re(\beta_2) \leqslant ... \leqslant Re(\beta_{2n}) < 0 < Re(\beta_{2n+1}) \leqslant Re(\beta_{2n+2}) \leqslant ... \leqslant Re(\beta_{4n}).$$

  Then, $Q^{(\hat{a},-)}$ has $2n$ distinct eigenvalues $\{\beta_i, i = 1, 2, ..., 2n\}$.

Numerical Method

Step 2 We compute γ_i by solving a system of linear equations $K(\beta_i)\gamma_i = 0$ and then

$$\vartheta_i = (\gamma_{i,n+1}, ..., \gamma_{i,3n})' \qquad i = 1, 2, .., 2n.$$

Step 3 We compute the $\zeta^{(\hat{a},-)}$ by solving a system of linear equations as follows:

$$Z' \zeta_k^{(\hat{a},-)} = (\gamma_{1,k}, ..., \gamma_{2n,k})' \qquad k = 1, 2, ..., n,$$

where $Z = [\vartheta_1, \vartheta_2, ..., \vartheta_{2n}]$.

Step 4 We compute the

$$Q^{(\hat{a},-)} = Z \operatorname{diag}\{\beta_1, \beta_2, ..., \beta_{2n}\} Z^{-1}.$$

and

$$e^{Q^{(\hat{a},-)}x} = Z \operatorname{diag}\{e^{\beta_1 x}, e^{\beta_2 x}, ..., e^{\beta_{2n} x}\} Z^{-1}.$$

- ϑ_i is the eigenvector of $Q^{(\hat{a},-)}$ corresponding to the eigenvalue β_i and
$$\gamma_i = W^{(\hat{a},-)} \vartheta_i = (\gamma_{i,1}, ..., \gamma_{i,3n})'.$$
-
$$W^{(\hat{a},-)} = \begin{pmatrix} \zeta^{(\hat{a},-)} \\ I_{2n} \end{pmatrix}.$$

Bank's Capital Structure

The capital structure of the bank is assumed to include equity, CoCo bonds, straight bonds, deposits and deposit insurance.

- Straight bonds: face value D_1, a continuous coupon rate c_1 and a maturity profile $\psi_1(t) = me^{-mt}$.
- Deposits: face value D_2, a continuous coupon rate c_2 and a maturity profile $\psi_2(t) = ke^{-kt}$.
- CoCos: face value D_3, continuous coupon rate paid before conversion c_3 and a maturity profile $\psi_3(t) = le^{-lt}$.
- Conversion time

  $$\tau_1 = \inf\{t \geqslant 0 : V_t < \alpha(D_1 + D_2 + D_3)\}$$

  and default time

  $$\tau_2 = \inf\{t \geqslant 0 : V_t < \alpha(D_1 + D_2)\}.$$

Straight Bonds

The market value of straight bonds with face value 1 maturing at t is expressed as follows

$$B(t) = E\left(\int_0^{\min(t,\tau_2)} e^{-\int_0^s r_u \mathrm{d}u} c_1 \mathrm{d}s + \mathbb{1}_{\{t \geqslant \tau_2\}} e^{-\int_0^{\tau_2} r_u \mathrm{d}u} \pi_1 + \mathbb{1}_{\{t < \tau_2\}} e^{-\int_0^t r_u \mathrm{d}u} \right).$$

Then, the total value of straight bonds is

$$B = D_1 \int_0^\infty m e^{-mt} B(t) \mathrm{d}t$$

so that

$$\begin{aligned} B &= (c_1 + m) D_1 \left(Y_0' W^{(\hat{r}+m,-)} e^{Q^{(\hat{r}+m,-)}(x - x_C)} \tilde{H}(0) - J_0' \right) \left(Q - \operatorname{diag}(\hat{r} + m) \right)^{-1} \mathbf{1} \\ &\quad + \pi_1 D_1 Y_0' W^{(\hat{r}+m,-)} e^{Q^{(\hat{r}+m,-)}(x - x_C)} \mathbf{1}. \end{aligned}$$

- π_1 is the constant recovery rate of bonds book value for debt holders after liquidation.
- $x_C = \ln \dfrac{\alpha(D_1 + D_2)}{V_0}$.
- $$\tilde{H}(w) = \begin{pmatrix} I_n \\ \begin{pmatrix} \frac{\hat{\eta}_{21}}{w + \hat{\eta}_{21}} & & \\ & \ddots & \\ & & \frac{\hat{\eta}_{2n}}{w + \hat{\eta}_{2n}} \end{pmatrix} \end{pmatrix}$$

Insured Deposits

The market value of deposits with face value 1 maturing at t is expressed as follows

$$D(t) = E\left(\int_0^{\min(t,\tau_2)} e^{-\int_0^s r_u \mathrm{d}u} c_2 \mathrm{d}s + \mathbb{1}_{\{t \geqslant \tau_2\}} e^{-\int_0^{\tau_2} r_u \mathrm{d}u} \pi_2 + \mathbb{1}_{\{t < \tau_2\}} e^{-\int_0^t r_u \mathrm{d}u} \right).$$

The total value of deposits is computed in the same way as that of straight bonds:

$$D = D_2 \int_0^\infty k e^{-kt} D(t) \mathrm{d}t$$

so that

$$\begin{aligned} D \quad &= (c_2 + k) D_2 \left(Y_0' W^{(\hat{r}+k,-)} e^{Q^{(\hat{r}+k,-)}(x - x_C)} \tilde{H}(0) - J_0' \right) \left(Q - \mathrm{diag}(\hat{r} + k) \right)^{-1} \mathbf{1} \\ &+ \pi_2 D_2 Y_0' W^{(\hat{r}+k,-)} e^{Q^{(\hat{r}+k,-)}(x - x_C)} I. \end{aligned}$$

- π_2 is the constant recovery rate of deposits book value for deposit holders after liquidation.

Deposit Insurance

The market value of deposit insurance is

$$DI = E\left(e^{-\int_0^{\tau_2} r_u \mathrm{d}u}\left[E\left(\int_0^{\infty} ke^{-kt}\left(\int_{\tau_2}^{\tau_2+t} e^{-\int_{\tau_2}^{s} r_u \mathrm{d}u} c_2 D_2 \mathrm{d}s + e^{-\int_{\tau_2}^{\tau_2+t} r_u \mathrm{d}u} D_2\right)\mathrm{d}t - \pi_2 D_2 \mid \mathcal{F}_{\tau_2}\right)\right]^{+}\right).$$

We have:

$$DI \quad = Y_0' W^{(r,-)} e^{Q^{(r,-)}(x - x_C)} \tilde{H}(0) \tilde{K}.$$

- The n vector

$$\tilde{K} = \begin{pmatrix} \left(-(c_2 + k) D_2 e_1{}' \left(Q - \mathrm{diag}(k + \hat{r})\right)^{-1} \mathbf{1} - \pi_2 D_2\right)^{+} \\ \vdots \\ \left(-(c_2 + k) D_2 e_n{}' \left(Q - \mathrm{diag}(k + \hat{r})\right)^{-1} \mathbf{1} - \pi_2 D_2\right)^{+} \end{pmatrix}.$$

CoCos

The market value of CoCo bonds with face value 1 maturing at t is

$$C(t) = E\left(\int_0^{\min(t,\tau_1)} e^{-\int_0^s r_u \mathrm{d}u} c_3 \mathrm{d}s + \frac{1-\rho}{D_3} \mathbb{1}_{\{\tau_1 \leqslant t\}} e^{-\int_0^{\tau_1} r_u \mathrm{d}u} S_{\tau_1} + \mathbb{1}_{\{\tau_1 > t\}} e^{-\int_0^t r_u \mathrm{d}u} \right).$$

Then, the value of all CoCo bonds is

$$C = D_3 \int_0^\infty l e^{-lt} C(t) \mathrm{d}t.$$

and

$$C = \underbrace{D_3(c_3 + l) E\left(\int_0^{\tau_1} e^{-\int_0^s (l+r_u) \mathrm{d}u} \mathrm{d}s \right)}_{C_1} + \underbrace{\left(1-\rho\right) E\left(e^{-\int_0^{\tau_1} (l+r_u) \mathrm{d}u} S_{\tau_1} \right)}_{C_2}.$$

- ρ is the proportion of original shareholders after conversion.

CoCos

We have:

$$
\begin{aligned}
C \quad &= D_3(c_3+l)\left(Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)} \tilde{H}(0) - J_0'\right)\left(Q - \operatorname{diag}(\hat{r}+l)\right)^{-1}\mathbf{1} \\
&+(1-\rho)V_0 e^{x_B} Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\tilde{I} \\
&+(1-\rho)\left(\gamma(c_1 D_1 + c_2 D_2) - \sigma D_2\right) Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\left(\tilde{M}_1 + \tilde{G}_1\tilde{H}(0) - \tilde{H}(0)\right) \\
&\quad\left(Q - \operatorname{diag}(\hat{r})\right)^{-1}\mathbf{1} \\
&+(1-\rho)Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\left(\tilde{M}_1 + \tilde{G}_1\tilde{H}(0)\right)\tilde{K} \\
&-(1-\rho)(1-\kappa)V_0 Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\left(\tilde{N}_2 + e^{x_C}\tilde{G}_1\tilde{I}\right) \\
&-(1-\rho)(c_1+m)D_1 Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\left(\tilde{M}_1 + \tilde{G}_2\tilde{H}(0) - \tilde{H}(0)\right)\left(Q - \operatorname{diag}(\hat{r}+m)\right)^{-1}\mathbf{1} \\
&-(1-\rho)\pi_1 D_1 Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\left(\tilde{G}_2\mathbf{1} - \tilde{G}_1\mathbf{1}\right) \\
&-(1-\rho)(c_2+k)D_2 Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\left(\tilde{M}_1 + \tilde{G}_3\tilde{H}(0) - \tilde{H}(0)\right)\left(Q - \operatorname{diag}(\hat{r}+k)\right)^{-1}\mathbf{1} \\
&-(1-\rho)\pi_2 D_2 Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)}\left(\tilde{G}_3\mathbf{1} - \tilde{G}_1\mathbf{1}\right).
\end{aligned}
$$

- $$x_B = \ln\frac{\alpha(D_1 + D_2 + D_3)}{V_0}$$

  .

CoCos

- $$\tilde{G}_1 = \begin{pmatrix} \begin{pmatrix} \left(s'_{n+1} W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x_B - x_C)}\right)' \\ \vdots \\ \left(s'_{2n} W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x_B - x_C)}\right)' \end{pmatrix} \\ \begin{pmatrix} \left(s'_{2n+1} W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x_B - x_C)} \hat{\eta}_{21} (Q^{(\hat{r},-)} + \hat{\eta}_{21} I)^{-1} \left(I - e^{(Q^{(\hat{r},-)} + \hat{\eta}_{21} I)(x_C - x_B)}\right)\right)' \\ \vdots \\ \left(s'_{3n} W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x_B - x_C)} \hat{\eta}_{2n} (Q^{(\hat{r},-)} + \hat{\eta}_{2n} I)^{-1} \left(I - e^{(Q^{(\hat{r},-)} + \hat{\eta}_{2n} I)(x_C - x_B)}\right)\right)' \end{pmatrix} \end{pmatrix},$$
- $2n \times n$ matrix

$$\tilde{N}_1 = \begin{pmatrix} O_n \\ \begin{pmatrix} e^{x_B} \frac{\hat{\eta}_{21}}{\hat{\eta}_{21}+1} e^{(\hat{\eta}_{21}+1)(x_C - x_B)} & & \\ & \ddots & \\ & & e^{x_B} \frac{\hat{\eta}_{2n}}{\hat{\eta}_{2n}+1} e^{(\hat{\eta}_{2n}+1)(x_C - x_B)} \end{pmatrix} \end{pmatrix},$$

CoCos

- $2n$ vector

$$\tilde{N}_2 = \begin{pmatrix} \mathbf{0}_n \\ \begin{pmatrix} e^{x_B} \frac{\hat{\eta}_{21}}{\hat{\eta}_{21}+1} e^{(\hat{\eta}_{21}+1)(x_C - x_B)} \\ \vdots \\ e^{x_B} \frac{\hat{\eta}_{2n}}{\hat{\eta}_{2n}+1} e^{(\hat{\eta}_{2n}+1)(x_C - x_B)} \end{pmatrix} \end{pmatrix},$$

- $2n \times n$ matrix

$$\tilde{M}_1 = \begin{pmatrix} O_n \\ \begin{pmatrix} e^{\hat{\eta}_{21}(x_C - x_B)} & & \\ & \ddots & \\ & & e^{\hat{\eta}_{2n}(x_C - x_B)} \end{pmatrix} \end{pmatrix},$$

- $2n$ vector

$$\tilde{M}_2 = \begin{pmatrix} \mathbf{0}_n \\ \begin{pmatrix} e^{\hat{\eta}_{21}(x_C - x_B)} \\ \vdots \\ e^{\hat{\eta}_{2n}(x_C - x_B)} \end{pmatrix} \end{pmatrix}.$$

- $\tilde{G}_2$ and $\tilde{G}_3$ are the $2n \times 2n$ matrices by replacing $W^{(\hat{r},-)}$, $Q^{(\hat{r},-)}$ by $W^{(\hat{r}+m,-)}$, $Q^{(\hat{r}+m,-)}$ and $W^{(\hat{r}+k,-)}$, $Q^{(\hat{r}+k,-)}$ in the $\tilde{G}_1$, respectively.

Bank Equity

The total bank value is

$$BV = V_0 + \gamma \underbrace{E\left(\int_0^{\tau_1} e^{-\int_0^s r_u \mathrm{d}u} c_3 D_3 \mathrm{d}s + \int_0^{\tau_2} e^{-\int_0^s r_u \mathrm{d}u} \left(c_1 D_1 + c_2 D_2 \right) \mathrm{d}s \right)}_{\text{tax benefits}} + DI$$

$$- \underbrace{E\left(e^{-\int_0^{\tau_2} r_u \mathrm{d}u} \left((1-\kappa) V_{\tau_2} - \pi_1 D_1 - \pi_2 D_2 \right) \right)}_{\text{bankruptcy costs}} - \underbrace{E(\int_0^{\tau_2} e^{-\int_0^s r_u \mathrm{d}u} \sigma D_2 \mathrm{d}s)}_{\text{deposit insurance premiums}} .$$

so that

$$\begin{aligned} BV &= V_0 + \gamma c_3 D_3 \left(Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x - x_B)} \tilde{H}(0) - J_0' \right) \left(Q - \mathrm{diag}(\hat{r}) \right)^{-1} \mathbf{1} \\ &+ (\gamma (c_1 D_1 + c_2 D_2) - \sigma D_2) \left(Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x - x_C)} \tilde{H}(0) - J_0' \right) \left(Q - \mathrm{diag}(\hat{r}) \right)^{-1} \mathbf{1} \\ &+ DI - (1-\kappa) V_0 e^{x_C} Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x - x_C)} \tilde{I} \\ &+ (\pi_1 D_1 + \pi_2 D_2) Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x - x_C)} \mathbf{1}. \end{aligned}$$

- κ is the constant recovery rate of asset value for the distribution to shareholders after liquidation.
- γ is the tax rate.

Bank Equity

Then, the market value of bank equity before conversion is

$$S = BV - B - C - D$$

and

$$\begin{aligned} S \quad = & \; V_0 + \gamma c_3 D_3 \left(Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x-x_B)} \tilde{H}(0) - J_0' \right) \left(Q - \mathrm{diag}(\hat{r}) \right)^{-1} \mathbf{1} \\ & + (\gamma (c_1 D_1 + c_2 D_2) - \sigma D_2) \left(Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x-x_C)} \tilde{H}(0) - J_0' \right) \left(Q - \mathrm{diag}(\hat{r}) \right)^{-1} \mathbf{1} \\ & - (1-\kappa) V_0 e^{x_C} Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x-x_C)} \tilde{I} \\ & + (\pi_1 D_1 + \pi_2 D_2) Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x-x_C)} \mathbf{1} \\ & + DI - B - C - D. \end{aligned}$$

Conversion and Default Probabilities

We have:

$$E\left(e^{-\kappa_1\tau_1}\right) = Y_0' W^{(\hat{\kappa}_1,-)} e^{Q^{(\hat{\kappa}_1,-)}(x-x_B)}\mathbf{1}$$

and

$$E\left(e^{-\kappa_2\tau_2}\right) = Y_0' W^{(\hat{\kappa}_2,-)} e^{Q^{(\hat{\kappa}_2,-)}(x-x_C)}\mathbf{1}$$

where $\mathbf{1} \in \mathbb{R}^{2n}$ is a vector of ones, $\hat{\kappa}_1 = (\kappa_1, ..., \kappa_1)'$ and $\hat{\kappa}_2 = (\kappa_2, ..., \kappa_2)'$. Denote $f_1(t) = P(\tau_1 \leqslant t)$ and $f_2(t) = P(\tau_2 \leqslant t)$. Their Laplace transforms satisfy:

$$\hat{f}_1(\kappa_1) = \int_0^\infty e^{-\kappa_1 t} P(\tau_1 \leqslant t)\mathrm{d}t = \frac{1}{\kappa_1}\int_0^\infty e^{-\kappa_1 t} dP(\tau_1 \leqslant t) = \frac{1}{\kappa_1} E\left(e^{-\kappa_1\tau_1}\right)$$

and

$$\hat{f}_2(\kappa_2) = \int_0^\infty e^{-\kappa_2 t} P(\tau_2 \leqslant t)\mathrm{d}t = \frac{1}{\kappa_2}\int_0^\infty e^{-\kappa_2 t} dP(\tau_2 \leqslant t) = \frac{1}{\kappa_2} E\left(e^{-\kappa_2\tau_2}\right).$$

We can compute the conversion and default probabilities $P(\tau_1 \leqslant t)$ and $P(\tau_2 \leqslant t)$ by numerically inverting the above Laplace transforms.

Market Yields and Premium Rate

We assume the straight bonds and CoCos are issued at par. Then, we can compute the coupon rate require by the market. We equate the market value of straight bonds B to the face value D_1 and have:

$$c_1 = \frac{1 - \pi_1 Y_0' W^{(\hat{r}+m,-)} e^{Q^{(\hat{r}+m,-)}(x-x_C)} \mathbf{1}}{\left(Y_0' W^{(\hat{r}+m,-)} e^{Q^{(\hat{r}+m,-)}(x-x_C)} \tilde{H}(0) - J_0' \right) \left(Q - \operatorname{diag}(\hat{r}+m) \right)^{-1} \mathbf{1}} - m.$$

In the same way, we have:

$$c_2 = \frac{1 - \pi_2 Y_0' W^{(\hat{r}+k,-)} e^{Q^{(\hat{r}+k,-)}(x-x_C)} \mathbf{1}}{\left(Y_0' W^{(\hat{r}+k,-)} e^{Q^{(\hat{r}+k,-)}(x-x_C)} \tilde{H}(0) - J_0' \right) \left(Q - \operatorname{diag}(\hat{r}+k) \right)^{-1} \mathbf{1}} - k$$

and

$$c_3 = \frac{D_3 - C_2}{D_3 \left(Y_0' W^{(\hat{r}+l,-)} e^{Q^{(\hat{r}+l,-)}(x-x_B)} \tilde{H}(0) - J_0' \right) \left(Q - \operatorname{diag}(\hat{r}+l) \right)^{-1} \mathbf{1}} - l.$$

Market Yields and Premium Rate

For debts issued at par, the aggregated yields equals the coupon rates and they depend on maturity through the parameters m, k, l, respectively. We therefore can interpret these coupon rates as market yields.

The aggregated credit spreads become $c_1 - r, c_2 - r, c_3 - r$ for the three types of debts.

We equate the market value of deposit insurance DI to the deposit insurance premium and have:

$$\sigma = \frac{Y_0' W^{(r,-)} e^{Q^{(r,-)}(x-x_C)} \tilde{H}(0) \tilde{K}}{D_2 \left(Y_0' W^{(\hat{r},-)} e^{Q^{(\hat{r},-)}(x-x_C)} \tilde{H}(0) - J_0' \right) \left(Q - \mathrm{diag}(\hat{r}) \right)^{-1} \mathbf{1}}.$$

- σ is a continuous deposit insurance premium rate to guarantee that the deposit will be fully paid if it is liquidated.

Illustration

With weights of loans and asset investment in the bank's balance sheet, we compute the market value of total assets for this bank through weighted sum of market value of different assets, which are indicated by corresponding daily market indices or computed from Euribor rates between January 3, 2000 and February 10, 2017.

Table: The balance sheet of a typical bank

	Proportion of Assets
Assets	
Loans	70%
Stocks	10%
Corporate Bonds	10%
Government Bonds	10%
Liabilities & Equity	
Deposits	60%
Classic Debt	20%
Equity	10%
CoCos	10%

Table: The market indices or proxy indices for the bank's assets

	Market Indices
The rate of Loans	2%+1 Year Euribor rate
Stocks	The Bloomberg European 500 Index
Corporate Bonds	The BofA Merrill Lynch AAA Euro Broad Market Index
	The BofA Merrill Lynch AA Euro Broad Market Index
	The BofA Merrill Lynch A Euro Broad Market Index
	The BofA Merrill Lynch BBB Euro Broad Market Index
Government Bonds	The BofA Merrill Lynch All Maturity All Euro Government Index

Illustration

We estimate the parameters of the models JD, RSBM and RSJD using the market value of total asset by performing a likelihood maximization.

Table: Parameter estimation for the JD, RSBM and RSJD

Model	Parameters					
	q_{12}	q_{21}	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\sigma}_1$	$\hat{\sigma}_2$
JD			0.0196		0.1436	
RSBM	0.0364	0.0119	0.0331	0.0645	0.3955	0.1788
RSJD	0.0267	0.0089	-0.0965	0.0418	0.3418	0.0108

$\hat{\lambda}_1$	$\hat{\lambda}_2$	p_1	p_2	$\hat{\eta}_{11}$	$\hat{\eta}_{21}$	$\hat{\eta}_{12}$	$\hat{\eta}_{22}$
1.2878		0.7370		9.8393		5.6439	
10.3973	11.9002	0.9754	0.5933	54.9706	30.8009	4.9990	23.2581

- The double exponential jump diffusion model (JD), the regime switching Brownian motion model (RSBM) and the regime switching double exponential jump diffusion model (RSJD).
- For the RSBM and RSJD, the sample likelihood is computed by using the Hamilton filter method in Hamilton (1989).
- For the JD and RSJD, the density of the distribution is not available in analytical form and the probability densities are computed by performing a Fourier inversion of the corresponding characteristic function.
- For simple statement, we only consider two regimes for RSBM and RSJD and the general n regimes case can be easily computed by changing 2×2 matrix into $n \times n$ matrix in computation.

Illustration

Table: Other parameters

r	0.0205
(D_1, D_2, D_3)	(20,60,10)
(c_1, c_2, c_3)	(r+2%,r+1%,r+3%)
(m, k, l)	(1/15,1,1/15)
τ	30%
δ	3%
ρ	90%
σ	1%
α	1
V_0	100
κ	10%
(π_1, π_2)	(85%,95%)
J_0	(1,0)'
Y_0	(0,0,0,0,1,0)'

- We use the BofA Merrill Lynch Euro Treasury Bill Index to compute the r.
- The Swiss Contingent Convertible Capital Proposal proposes the 19% total capital ratio can be composed of 9% contingent convertible capital and therefore we choose 10% for the proportion of CoCos in this setting.
- Some other parameters setting refer to Glasserman and Nouri (2012) and Nouri (2012).

Illustration

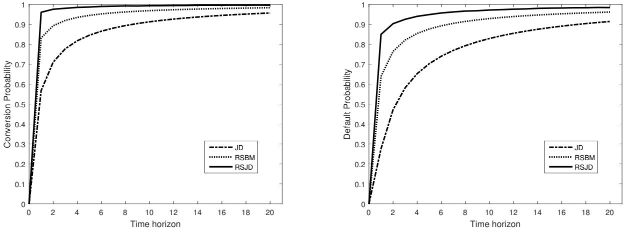
The result shows the RSJD achieves the best fit. The comparison among these models indicates that there exists significant phenomenon of regime switching and the characterization of regime switching risk is more important than that of jump risk.

Table: Loglikelihoods, AIC and BIC for the JD, RSBM and RSJD

	JD	RSBM	RSJD
LogLik	70.8341	293.2333	322.0280
AIC	-129.6681	-574.4666	-616.0560
BIC	-91.3730	-536.1715	-526.7007

Illustration

The Figure shows the characterization of regime switching risk increases a lot in the conversion and default probabilities. From an intuitive understanding, the high risk and low risk are compromised when the regime switching is not characterized, which results in lowering the conversion and default probabilities. The conversion and default probabilities of RSJD are higher than that of RSBM for the presence of jumps.



(a) (b)

Figure: (a) Conversion Probability (b) Default Probability

Illustration

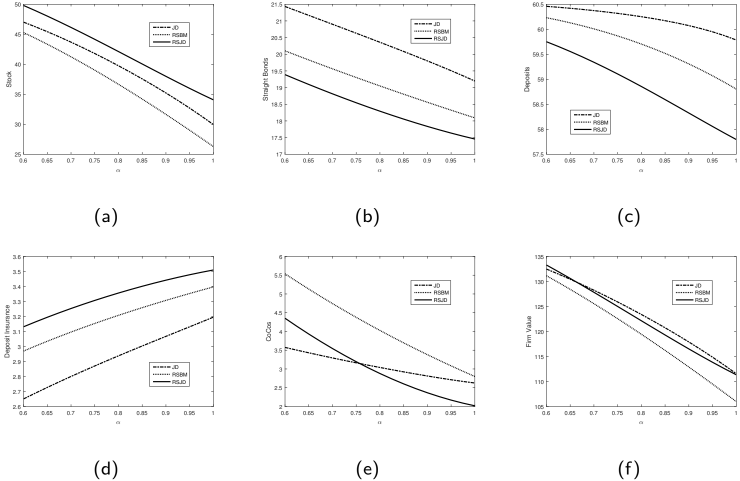
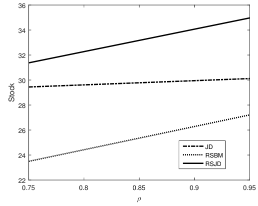
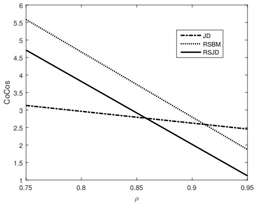


Figure: (a) Stock (b) Straight Bonds (c) Deposits
(d) Deposit Insurance (e) CoCos (f) Firm Value

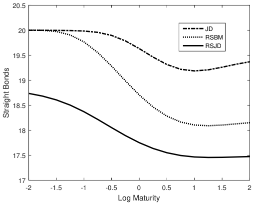
Illustration



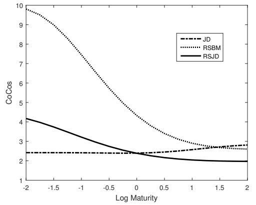
(a)



(b)



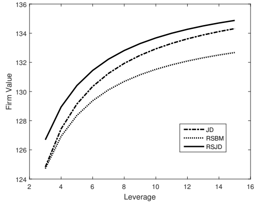
(c)



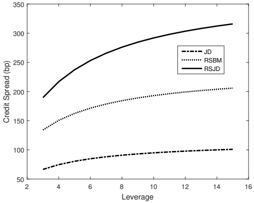
(d)

Illustration

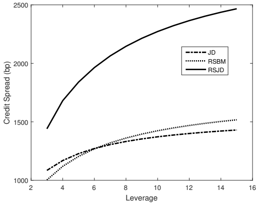
We make $\alpha = 0.6$ to take some investigations in a loose regulation environment.



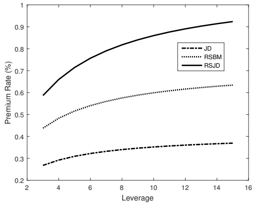
(e)



(f)

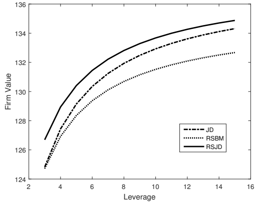


(g)

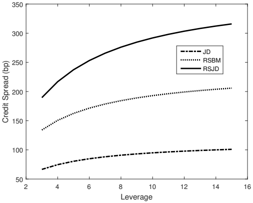


(h)

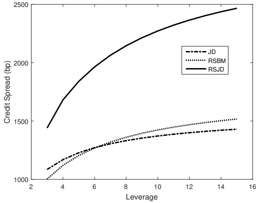
Illustration



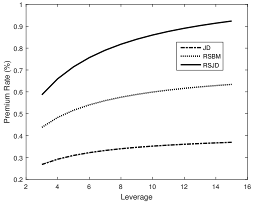
(i)



(j)



(k)



(l)