On Credit and Surrender Risks in Insurance Companies

A joint work by

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Outline of the Talk

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4. Some Theoretical Results
5. Case Study

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Life Office

Assets	Liabilities
A_0	$E_0 = (1-\alpha)A_0$ $L_0 = \alpha A_0$

– E_0 = initial equity value

– L_0 = initial policyholder investment

Participating Contracts
–> Minimum Guarantee

Existence of a minimum guaranteed rate r_g :

$$L_T^g = L_0 \, e^{r_g T} \quad \text{at } T$$

- **Solvency at time T : $\mathbf{A_T \geq L_T^g}$**

  Policyholders receive L_T^g

- **Default at time T : $\mathbf{A_T < L_T^g}$**

  Policyholders receive A_T

Participating Contracts
–> Participation Bonus

Bonus = δ times Benefits of the Company, when :

$$A_T > \frac{L_T^g}{\alpha} > L_T^g \qquad \left(\alpha = \frac{A_0}{L_0} < 1\right)$$

Assuming *no prior bankruptcy*, policyholders receive at T :

$$\Theta_L(T) = \begin{cases} A_T & \text{if } A_T < L_T^g \\ L_T^g & \text{if } L_T^g \le A_T \le \frac{L_T^g}{\alpha} \\ L_T^g + \delta(\alpha A_T - L_T^g) & \text{if } A_T > \frac{L_T^g}{\alpha} \end{cases}$$

Company Early Default

The firm pursues its activities until T iff :

$$\forall t \in [0, T[\quad , \quad A_t > L_0 e^{r_g t} \triangleq B_t$$

Let τ be the default time

$$\tau = \inf\{t \in [0, T] \ / \ A_t < B_t\}$$

In case of prior insolvency, policyholders receive :

$$\Theta_L(\tau) = \ A_\tau$$

(not necessarily equal to $L_0 e^{r_g \tau}$)

Modeling Surrenders

Let $N_t^i = 1_{\{\tau^i \leq t\}}$ be the

indicator function of surrender
of policyholder "i"

The number of cumulative surrenders is :

$$N_t = \sum_{i=1}^{I} N_t^i$$

where I is the initial number of policyholders

We also define by :

$$\bar{A}_t = \frac{A_t}{I - N_t}$$

the assets per remaining policyholder

Assumed Dynamics

The asset dynamics is
under the risk-neutral probability :

$$dA_t = A_{t-}\left(r_t dt + \sigma_t dW_t^Q - \frac{dN_t}{I - N_{t-}}\right)$$

The guarantee L_t is modeled accordingly :

$$dL_t = L_{t-}\left(\rho_g dt - \frac{dN_t}{I - N_{t-}}\right)$$

Assumed Dynamics

The interest rate dynamics is
under the risk-neutral probability :

$$dr_t = a\ (b - r_t)\ dt + \delta_t\ dZ_t^Q$$

The surrender intensity is supposed to follow :

$$d\lambda_t = c\ (d - \lambda_t)\ dt + \epsilon_t\ dX_t$$

where we assumed W, Z and X correlated

We also imposed a Gaussian copula
on surrenders
(common parameter ρ)

Balance Sheet Interpretations

The balance sheet of the firm is, before a surrender event :

$t-$	$I - N_{t-}$
$(I - N_{t-})\bar{A}_t$	$(I - N_{t-})\bar{S}_t$
	$(I - N_{t-})\bar{L}_t$

Balance Sheet Interpretations

Then, after a surrender event :

t	$I - N_t$
$(I - N_t)\bar{A}_t$	$(I - N_t)\bar{S}_t$
	$(I - N_t)\bar{L}_t$

Theoretical Results

It can be shown that :

$$A_t = A_0 \exp\left(\int_0^t \left(r_u - \frac{\sigma_u^2}{2}\right) du + \int_0^t \sigma_u dW_u^Q\right) \frac{I - N_t}{I}$$

$$L_t = L_0 e^{\rho_g t} \frac{I - N_t}{I} = (I - N_t)\bar{L}_T e^{-\rho_g (T-t)}$$

$$\bar{A}_t = \bar{A}_0 \ \exp\left(\int_0^t \left(r_u - \frac{\sigma_u^2}{2}\right) du + \int_0^t \sigma_u dW_u^Q\right)$$

Assumed Dynamics
An Alternative Framework

The asset dynamics is
under the risk-neutral probability :

$$dA_t = A_{t-} \left(r_t dt + \sigma_t dW_t^Q - \frac{1}{A_{t-}} \frac{\beta L_{t-}}{I - N_{t-}} dN_t \right)$$

The guarantee L_t is modeled accordingly :

$$dL_t = L_{t-} \left(\rho_g dt - \frac{dN_t}{I - N_{t-}} \right)$$

Balance Sheet Interpretations

The balance sheet of the firm is, before a surrender event :

$t-$	$I - N_{t-}$
$(I - N_{t-})\bar{A}_{t-}$	$(I - N_{t-})\bar{S}_{t-}$ $(I - N_{t-})\bar{L}_t$

Balance Sheet Interpretations

Then, after a surrender event :

t	$I - N_t$
$(I - N_t)\bar{A}_{t-}$ $+(\bar{A}_{t-} - \beta\bar{L}_t)$	$(I - N_t)\bar{S}_{t-}$ $+(\bar{A}_{t-} - \beta\bar{L}_t)$ $(I - N_t)\bar{L}_t$

➠ A high β means surrenders contribute to default

Theoretical Results

It can be shown that :

$$A_t = A_0 \exp\left(\int_0^t \left(r_u - \frac{\sigma_u^2}{2}\right) du + \int_0^t \sigma_u dW_u^Q\right)$$
$$- \int_0^t \exp\left(\int_s^t \left(r_u - \frac{\sigma_u^2}{2}\right) du + \int_s^t \sigma_u dW_u^Q\right) \beta \bar{L}_s dN_s$$

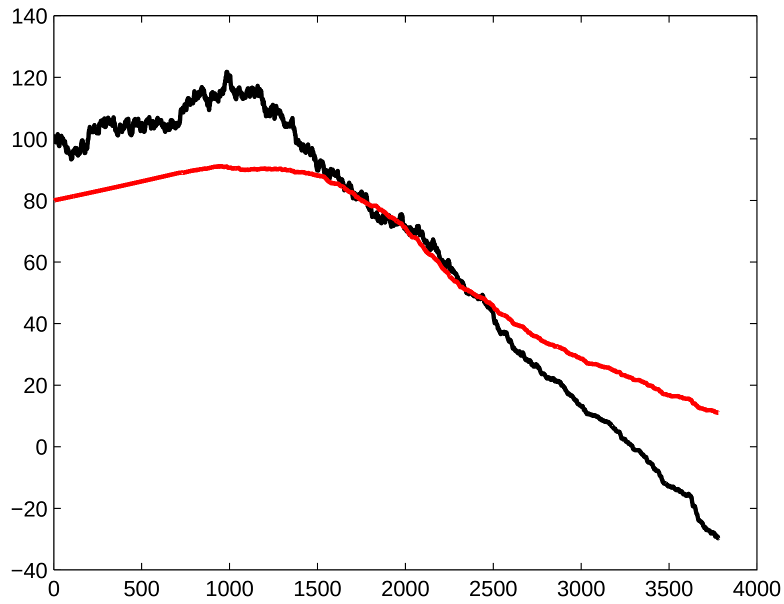
$$L_t = L_0 e^{\rho_g t} \frac{I - N_t}{I} = (I - N_t) \bar{L}_T e^{-\rho_g (T-t)}$$

An additional assumption

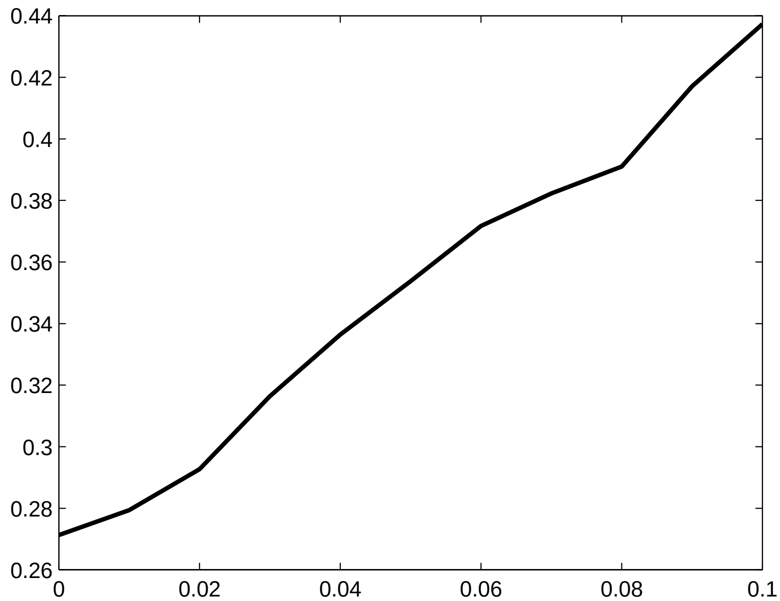
We allow for bubble/acceleration effects as :

$$\beta_t = \max\left(1, 1 + \frac{100 - A_t}{100}\right)$$

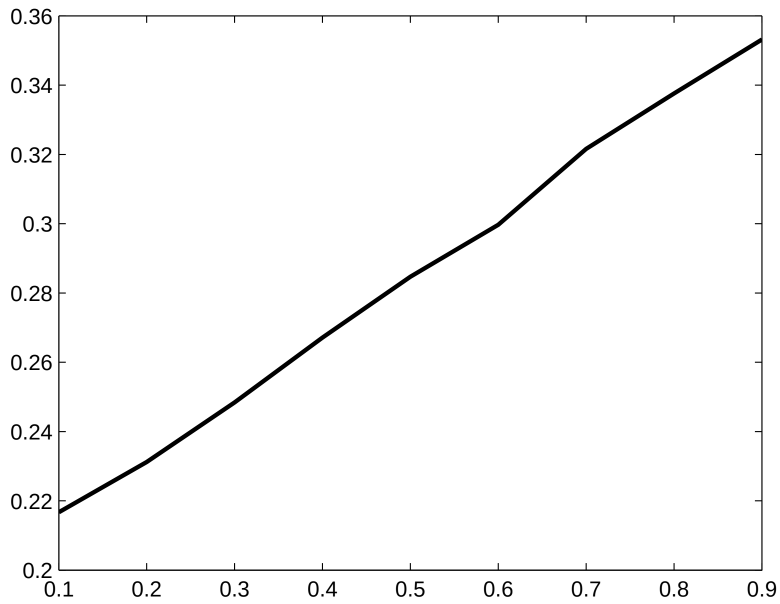
Example of a Ruin



Ruin Probability
w.r.t. Surrender Intensity



Ruin Probability
w.r.t. Surrender Correlation



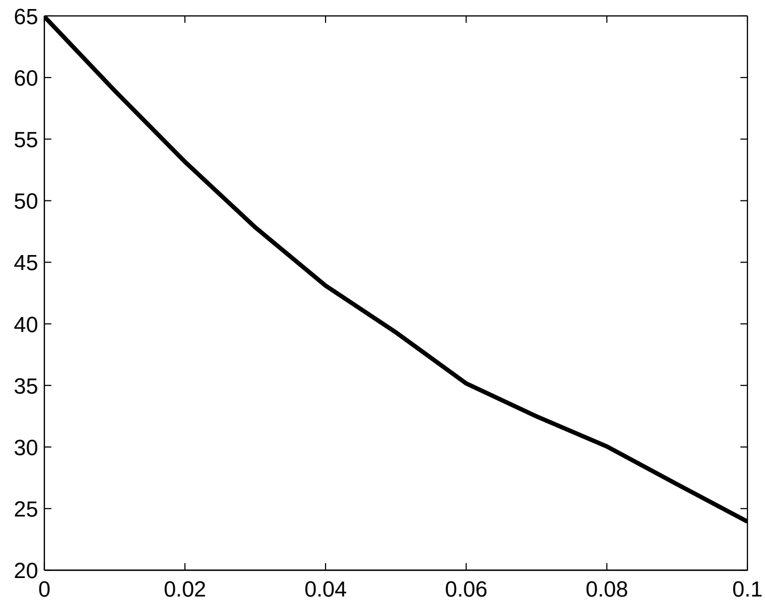
Value of Remaining Contracts

For policyholders remaining
with the firm until T,
and a firm which does not default :

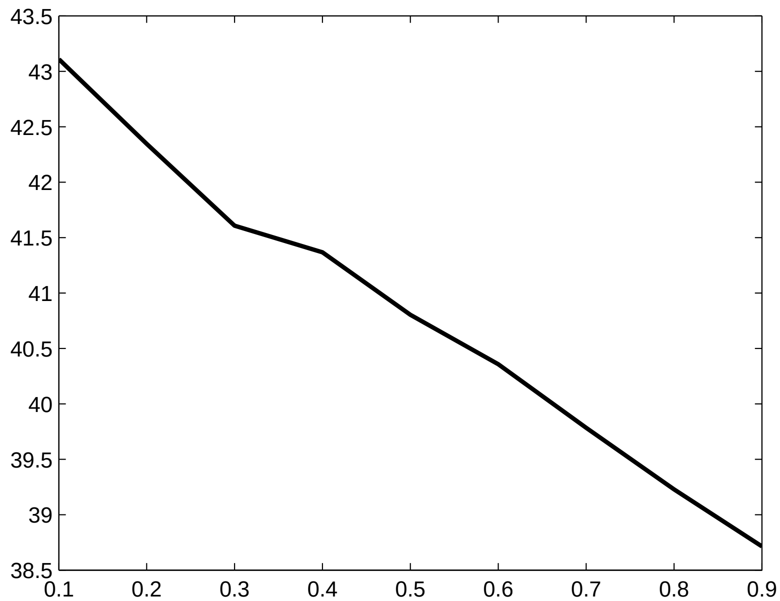
$$E^Q\left[(I-N_T)(1-M_T)e^{-\int_0^T r_u du}\left(\bar{L}_T+\delta(\alpha\bar{A}_T-\bar{L}_T)^+\right)\right]$$

where $M_t = 1_{\{\tau_d \leq t\}}$ is the indicator of default

Remaining Contracts Value w.r.t. Surrender Intensity



Remaining Contracts Value w.r.t. Surrender Correlation

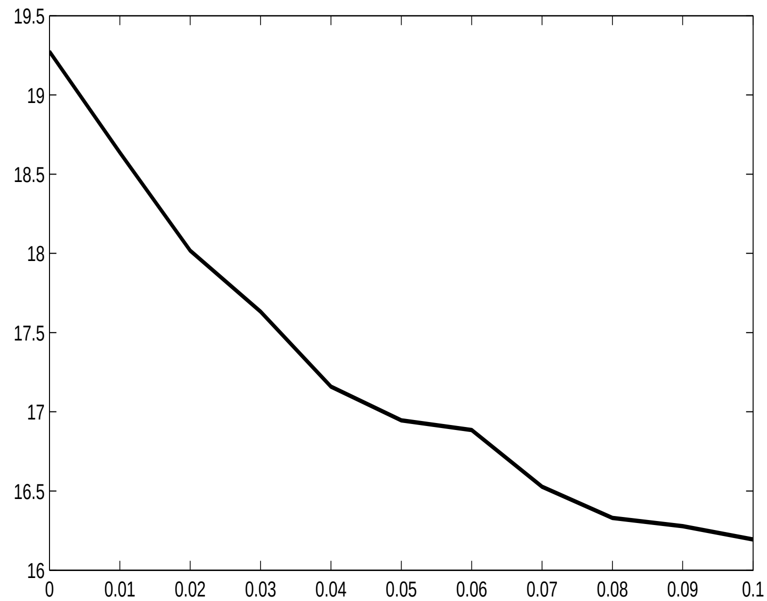


Value in Case of Default

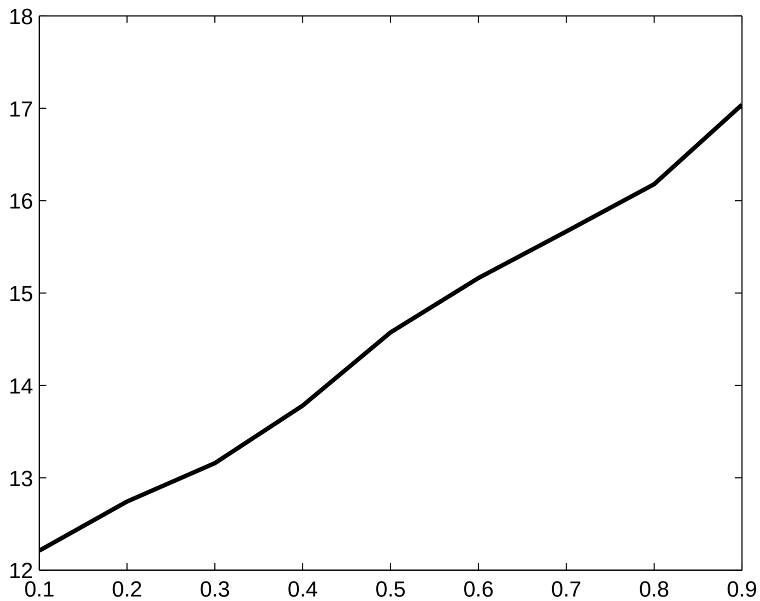
The value of the firm upon default is :

$$E^Q\left[\int_0^T (I - N_s)e^{-\int_0^s r_u du}\bar{A}_s dM_s\right]$$

Value in Case of Default w.r.t. Surrender Intensity



Value in Case of Default w.r.t. Surrender Correlation

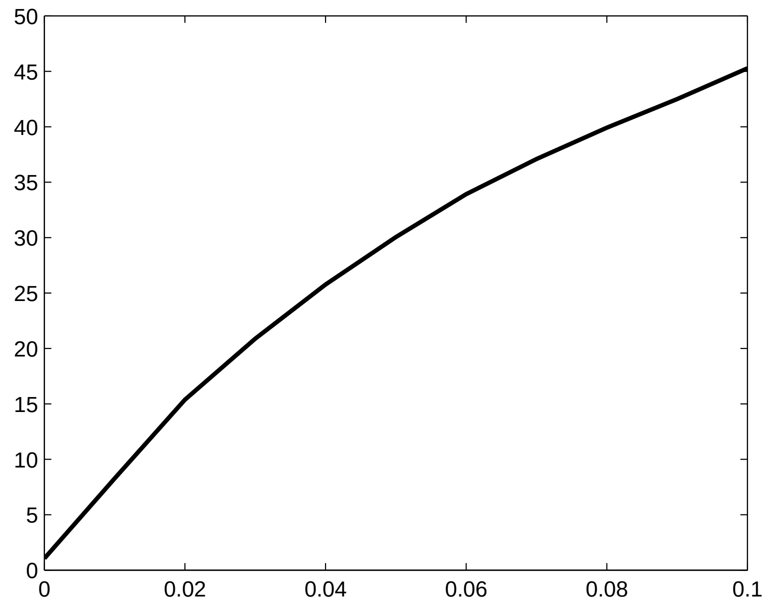


Value of Surrendered Contracts

The value of surrendered contracts is :

$$E^Q\left[\int_0^T (1 - M_{s-})e^{-\int_0^s r_u du}\bar{C}_{s-}dN_s\right]$$

Value of Surrendered Contracts w.r.t. Surrender Intensity



Value of Surrendered Contracts w.r.t. Surrender Correlation