An Index of (Absolute) Correlation Aversion
Theory and Some Implications

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A joint work with

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Outline of the Talk

– Bibliography
– Preliminary results :
  background risk and decreasing downside risk aversion
– In this study, we look at the relation between :
  cross background risks
  and
  (decreasing ?) cross downside risk aversion

Question under Study

So, we ask :
For what type of agent does
an additional zero-mean risk on *health*
induce a decrease of the investment in the risky part
of the portfolio constituting *wealth*?

Bibliography

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– Kimball (Econometrica, 1990)
– Gollier, Pratt (Econometrica, 1996)
– Eeckhoudt, Gollier, Schlesinger (Econometrica, 1996)
– Gollier (MIT, 2001)
– Courbage (Theory and Decision, 2001)
– Eeckhoudt, Rey, Schlesinger (MS, 2007)
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– Crainich, Eeckhoudt, Le Courtois (JME, 2014, forthcoming)

Preliminary Results

We first look for conditions ensuring that, for $E(\tilde{x}) = 0$:

$$\left[E(\tilde{y}u'(z+\tilde{y})) = 0 \Rightarrow E(\tilde{y}u'(z+\tilde{y}+\tilde{x})) \leq 0 \right]$$

In plain words, what are the conditions on the utility u such that the introduction of a so-called background risk $\tilde{x}$ to a portfolio made of a risk-free asset z and a risky asset $\tilde{y}$ reduces the proportion invested in the risky asset ?

Preliminary Results

The concept of Downside Risk Aversion, or DRA, is related to the quantity u'''/u' where

$$m = \frac{k}{2}\sigma(\tilde{\epsilon})^2 \frac{u'''(w)}{u'(w)}$$

solves in the small

$$\frac{1}{2}\left[u(w-k) + E[u(w+\tilde{\epsilon})]\right] = \frac{1}{2}\left[E[u(w-k+\tilde{\epsilon})] + u(w+m)\right]$$

So, m is the quantity that compensates the pain attached to the lottery that combines bad $(-k)$ with bad $(\tilde{x})$, compared to the lottery that combines good with bad.

Preliminary Results

Necessary condition for the background risk result :

$$\left[E(\tilde{y}u'(z+\tilde{y})) = 0 \Rightarrow E(\tilde{y}u'(z+\tilde{y}+\tilde{x})) \leq 0\right] \Rightarrow \text{DDRA}$$

where DDRA is

$$\forall w \quad \frac{\partial}{\partial w}\left(\frac{u'''(w)}{u'(w)}\right) \leq 0$$

Preliminary Results

Sufficient condition for the background risk result :

$$\left[E(\tilde{y}u'(z+\tilde{y})) = 0 \Rightarrow E(\tilde{y}u'(z+\tilde{y}+\tilde{x})) \leq 0\right] \Leftarrow \text{ Ross-DDRA}$$

where Ross-DDRA is

$$\forall t \ \forall w \quad \frac{\partial}{\partial w}\left(\frac{u'''(t+w)}{u'(w)}\right) \leq 0$$

Preliminary Results

Remark 1 :

$$\forall t \ \frac{u'''(t+.)}{u'(.)} \searrow \Leftrightarrow \ \exists \lambda \mid \forall w \ T(w) \geq \lambda \geq A(w)$$

where T and A are the temperance and risk aversion coefficients.

Preliminary Results

Remark 2 : the results are derived using
the diffidence theorem, stating that $\forall \tilde{x}$ of bounded support

$$E\left(f_1(\tilde{x})\right) = 0 \Rightarrow E\left(f_2(\tilde{x})\right) \leq 0$$

is equivalent to

$$\forall x \in [a, b] \quad f_2(x) \leq \frac{f_2'(x_0)}{f_1'(x_0)} f_1(x)$$

provided

- $\exists x_0 \mid f_1(x_0) = f_2(x_0) = 0$
- f_1 and f_2 are twice differentiable at x_0
- $f_1'(x_0) \neq 0$

Cross Background Risks and DRA

We first look for conditions ensuring that, for $E(\tilde{x}) = 0$:

$$E[u_1(z + (\tilde{y} - i), h)\tilde{y}] = 0 \Rightarrow E[u_1(z + (\tilde{y} - i), h + \tilde{x})\tilde{y}] \leq 0$$

In plain words, what are the conditions on the utility u such that the introduction of a so-called background risk $\tilde{x}$ on health (initial level : h) to a DM initially endowed with a portfolio made of a risk-free asset z and a risky asset $\tilde{y}$ reduces the proportion invested in the risky asset ?

Or, ‘do vapoteurs invest less in stocks ?’

Cross Background Risks and CDRA

The concept of Cross Downside Risk Aversion, or CDRA, is related to the quantity u_{122}/u_1 where

$$\widehat{m} = \frac{l}{4}\sigma(\tilde{\epsilon})^2 \frac{u_{122}(x,y)}{u_1(x,y)}$$

solves in the small

$$\frac{1}{2}\left[u(x-l,y) + E\left[u(x,y+\tilde{\epsilon})\right]\right] = \frac{1}{2}\left[E\left[u(x-l,y+\tilde{\epsilon})\right] + u(x+\widehat{m},y)\right]$$

So, $\widehat{m}$ compensates the pain attached to the lottery that combines bad on wealth $(-l)$ with bad on health $(\tilde{\epsilon})$, compared to the lottery that combines good with bad.

Cross Background Risks and CDRA

Necessary condition for the background risk result :

$$[E[u_1(z+(\tilde{y}-i),h)\tilde{y}]=0 \Rightarrow E[u_1(z+(\tilde{y}-i),h+\tilde{x})\tilde{y}] \leq 0] \;\Rightarrow\; \text{DCDRA}$$

where DCDRA is

$$\forall(s,t) \quad \frac{\partial}{\partial s}\left(\frac{u_{122}(s,t)}{u_1(s,t)}\right) \leq 0$$

Cross Background Risks and CDRA

Sufficient condition for the background risk result :

Ross-DCDRA $\Rightarrow$

$$[E[u_1(z+(\tilde{y}-i),h)\tilde{y}]=0 \Rightarrow E[u_1(z+(\tilde{y}-i),h+\tilde{x})\tilde{y}] \leq 0]$$

where Ross-DCDRA is

$$\forall (s,t,u) \quad \frac{\partial}{\partial s}\left(\frac{u_{122}(s,t+u)}{u_1(s,t)}\right) \leq 0$$

Alternative Approach

Remark :

$$\text{Ross-DCDRA} \Leftrightarrow \forall x \ \exists \lambda_x \ \forall y \mid -\frac{u_{1122}(x,y)}{u_{122}(x,y)} \geq \lambda_x \geq -\frac{u_{11}(x,y)}{u_1(x,y)}$$

where A and T are the risk aversion and cross-temperance coefficients.

In plain words, cross-temperance should always be superior to risk aversion for the background risk result to prevail.

Conclusion

To extend the results from Vapoteurs to Smokers, one needs to additionally assume that u_{12}/u_1 is decreasing in wealth.