Ruin Theory with Stable and Double Exponential Lévy Processes

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Outline of the Talk

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4. Ruin Probabibilities with "Kou" Processes
5. Empirical Relevance of our Approach
6. Conclusion

Bibliography

Perturbative Approaches :

Dufresne and Gerber (1991), Furrer (1998)

Direct Gamma Process Modelling :

Dufresne, Gerber and Shiu (1991)

Asymptotic Probabilities
with General Lévy Processes :

Klüppelberg, Kyprianou and Maller (2003)

Financial Default Probabilities
with Spectrally Negative Lévy Processes :

Hilberink and Rogers (2002)

The Problem at Stake

Most Models of Ruin Predict since Cramer and Lundberg :

$$P(\tau < +\infty) < 1$$

where τ is the default time.

On the contrary, we take a look at
two classes of models where

$$P(\tau < +\infty) = 1$$

We Justify our Approach both Theoretically and Empirically

Ruin Probabilities with Stable Lévy Processes

We consider processes with stationary independent increments that are stable, so with the characteristic function :

$$E\left(e^{iuX_1}\right) = e^{i\mu u - \sigma^\alpha |u|^\alpha [1 - i\beta\epsilon(u)\tan(\frac{\pi\alpha}{2})]}$$

where ϵ is the sign function.

In most circumstances (exclusion of subordinators), these processes satisfy :

$$P(\tau < +\infty) = 1$$

Ruin Probabilities with Stable Lévy Processes

The paper shows that finite-time
ruin probability can be expressed as :

$$R(u,t) = 1 - \mathcal{L}_q^{-1} \otimes \mathcal{L}_z^{-1} \left[\frac{\psi_{SL}^{-}(q,z)}{qz} \right]$$

where ψ^- is the Wiener-Hopf factor of the infimum :

$$\psi^-(q,z) = q \int_0^{+\infty} e^{-qt} E\left(e^{zI_t}\right) dt$$

so where one is left with the determination of ψ^-.

Ruin Probabilities with Stable Lévy Processes

Closed-form expressions for Wiener-Hopf factors are rare (BM, random walk, SN, Kou).

Doney (1987) obtained for Stable Lévy Processes :

$$\psi^{-}_{SL}(1,z) = \frac{\prod_{r=0}^{l-1}\left((-1)^{k+1}z + e^{i(l-1-2r)\pi/\alpha}\right)}{\prod_{r=0}^{k}\left((-1)^{l}z^{\alpha} + e^{i\alpha(k-2r)\pi}\right)}$$

Ruin Probabilities with Stable Lévy Processes

In the above function, k and l are solutions of $\rho + k = \frac{l}{\alpha}$ under the following definition of ρ :

$$\rho = \frac{1}{2} + \frac{1}{\pi\alpha}\tan^{-1}\left(\beta\tan\left(\frac{\pi\alpha}{2}\right)\right)$$

From $\psi^{-}_{SL}(1,z)$, we can deduce $\psi^{-}_{SL}(q,z)$ thanks to the following relationship :

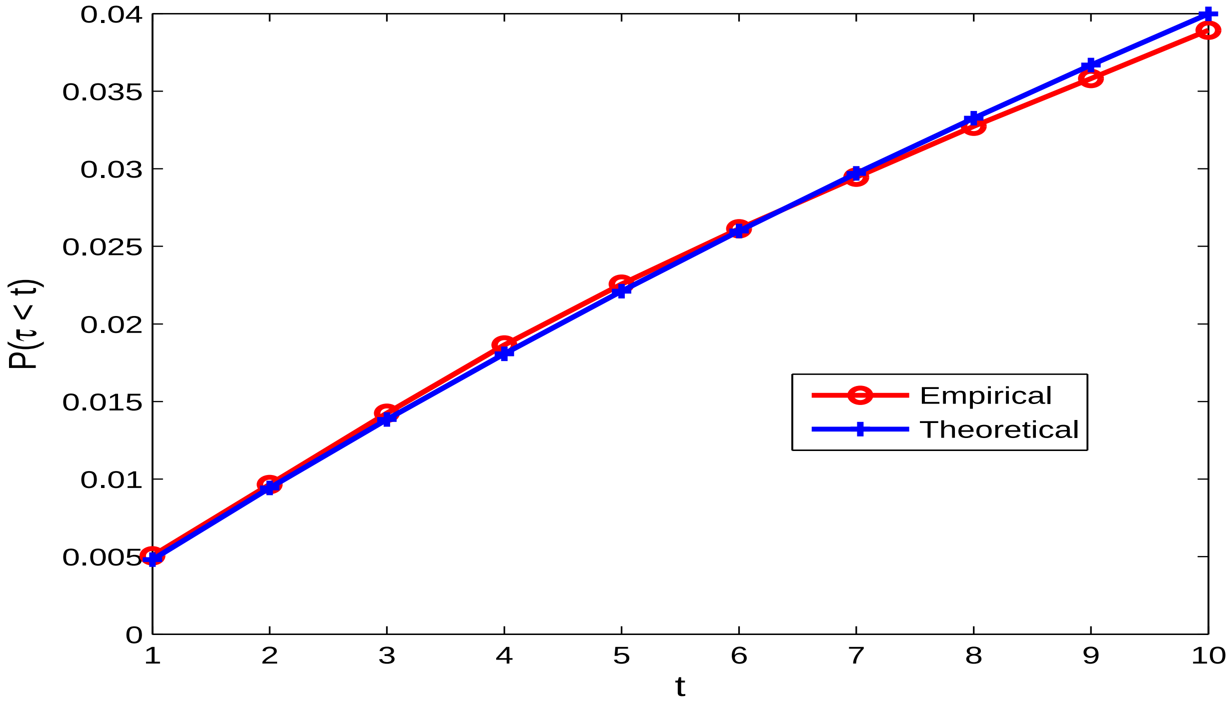
$$\psi^{-}_{SL}(q,z) = \psi^{-}_{SL}\left(1, zq^{-\frac{1}{\alpha}}\right)$$

Ruin Probabilities with Stable Lévy Processes

To estimate the ruin probability of an insurance company, one must therefore :

- ➠ Calibrate α, β and u on the data
- ➠ Obtain k and l
- ➠ Double Laplace Invert the Wiener-Hopf factor (function of k, l)

Ruin Probabilities with Stable Lévy Processes
Empirical Fit



Ruin Probabilities with Stable Lévy Processes
Conclusion

- Theoretical Ruin Probabilities fit the Empirical Data of Moody's
- This is all the more interesting that only 2 parameters are fitted (α and β)
- Sure Ruin at Infinity is Compatible with Realistic Finite-Time Ruin Probabilities

Ruin Probabilities with Kou Processes

The so-called Kou process is a jump diffusion defined by :

$$X_t = a\,t + \sigma z_t + \sum_{k=1}^{N_t} Y_k$$

where the law of jumps is doubly exponential :

$$f_Y(y) = p\,\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + (1-p)\,\eta_2 e^{\eta_2 y} 1_{\{y < 0\}}$$

Due to this law, many results nearby barriers were obtained.

Ruin Probabilities with Kou Processes

There are a few ways of computing finite-time ruin probabilities with these processes.

One can for instance use the Wiener-Hopf factor :

$$\psi^{-}_{KJ}(q,z) = \frac{\beta_{3,q}}{\beta_{3,q}+z}\;\frac{\beta_{4,q}}{\beta_{4,q}+z}\;\frac{\eta_2+z}{\eta_2}$$

where $\beta_{3,q}$ and $\beta_{4,q}$ are solutions of :

$$G(x) = q$$

where G is defined by :

$$G(x) = ax + \frac{1}{2}\sigma^2 x^2 + \lambda\left(p\frac{\eta_1}{\eta_1 - x} + (1-p)\frac{\eta_2}{\eta_2 + x} - 1\right)$$

Ruin Probabilities with Kou Processes

An important property of Kou processes is the following :

Define

$$\delta = a + \lambda \left(\frac{p}{\eta_1} - \frac{1-p}{\eta_2} \right)$$

then

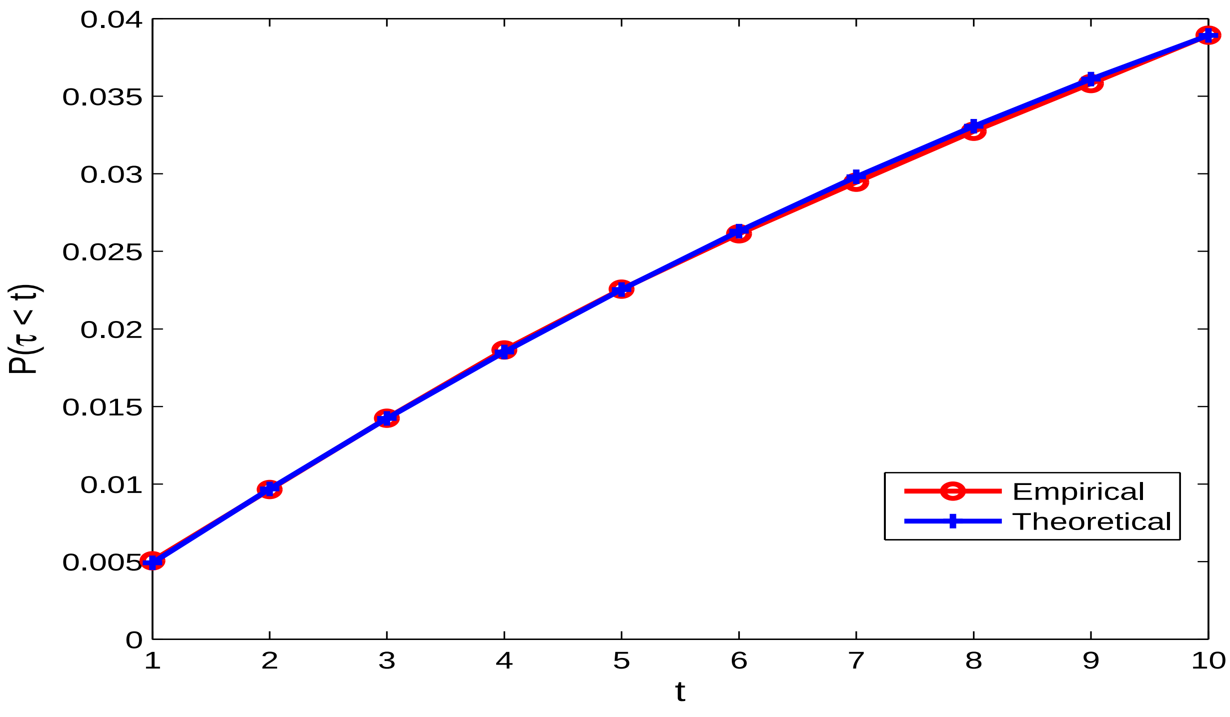
$$P(\tau_b < \infty) = 1, \text{ if } \delta < 0$$

and :

$$P(\tau_b < \infty) < 1, \text{ if } \delta \geq 0$$

Ruin Probabilities with Kou Processes

Empirical Fit



Ruin Probabilities with Kou Processes
Conclusion

- Again, Theoretical Ruin Probabilities fit the Empirical Data of Moody's
- More Interesting we obtain from the fit : $\delta \approx -0.5 < 0$
- This is a confirmation of $P(\tau_b < \infty) = 1$.

Conclusion

An Argument Often Opposed to Us :

"Why would I invest in a company that will go bankrupt ?"

Our answer :

"We want to invest in a company whose manager makes sure the probability of ruin in the coming 10 years remains small. Benefits in 10 million years are irrelevant to us."

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Common Sense