# Asset Risk Management of Participating Contracts

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## Outline of the Talk

1. Bibliography
2. Example of Contract
3. Asset Management
4. Illustration

## Bibliography


- Brennan and Schwartz [JOF, 1976]
- Briys and de Varenne [Wiley, 2001]
- Grosen and Jørgensen [JRI, 2002]
- Ballotta [IME, 2005]
- Bacinello [ASTIN Bulletin, 2001]

# Bibliography


- Longstaff and Schwartz [JOF, 1995]
- Collin-Dufresne and Goldstein [JOF, 2001]
- Bernard, Le Courtois and Quittard-Pinon [IME, 2005]
- Bernard, Le Courtois and Quittard-Pinon [NAAJ, 2006]
- Black, Perold [JEDC, 1992]

## Life Office

| Assets | Liabilities |
|---|---|
| $A_0$ | $E_0 = (1 - \alpha)A_0$<br>$L_0 = \alpha A_0$ |

– $E_0$ = initial equity value

– $L_0$ = initial policyholder investment

## Participating Contracts
## –> Minimum Guarantee

Existence of a minimum guaranteed rate $r_g$ :

$$L_T^g = L_0 \, e^{r_g T} \quad \text{at } T$$

➠ **Solvency at time $T$ : $\mathbf{A_T \geq L_T^g}$**

Policyholders receive $L_T^g$

➠ **Default at time $T$ : $\mathbf{A_T < L_T^g}$**

Policyholders receive $A_T$

## Participating Contracts
## –> Participation Bonus

Bonus $=\delta$ times Benefits of the Company, when :

$$A_T > \frac{L_T^g}{\alpha} > L_T^g \qquad \left(\alpha = \frac{A_0}{L_0} < 1\right)$$

Assuming *no prior bankruptcy*, policyholders receive at $T$ :

$$\Theta_L(T) = \begin{cases} A_T & \text{if } A_T < L_T^g \\ L_T^g & \text{if } L_T^g \leq A_T \leq \frac{L_T^g}{\alpha} \\ L_T^g + \delta(\alpha A_T - L_T^g) & \text{if } A_T > \frac{L_T^g}{\alpha} \end{cases}$$

## Company Early Default

The firm pursues its activities until $T$ iff :

$$\forall t \in [0, T[ \quad , \quad A_t > L_0 e^{r_g t} \triangleq B_t$$

Let $\tau$ be the default time

$$\tau = \inf\{t \in [0, T] \ / \ A_t < B_t\}$$

In case of prior insolvency, policyholders receive :

$$\Theta_L(\tau) = \ L_0 e^{r_g \tau}$$

# Asset Dynamics

The asset dynamics under the risk-neutral probability Q are :

$$\frac{dA_t}{A_t} = r_t dt + \sigma dZ^{\mathrm{Q}}(t)$$

Because a big proportion of the assets are made of bonds, an interest rate model is necessary.

$Z^{\mathrm{Q}}$ of the assets will be correlated to $Z_1^{\mathrm{Q}}$ of the interest rates ($dZ^{\mathrm{Q}}.dZ_1^{\mathrm{Q}} = \rho dt$).

## Stochastic Interest Rates

The dynamics under Q of the interest rate $r$ and the zero-coupon bonds $P(t,T)$ are :

$$dr_t = a(\theta - r_t)dt + \nu dZ_1^{\mathsf{Q}}(t)$$

and : $$\frac{dP(t,T)}{P(t,T)} = r_t dt - \sigma_P(t,T) dZ_1^{\mathsf{Q}}(t)$$

We Assume an Exponential Volatility for the Zero-Coupons :

$$\sigma_P(t,T) = \frac{\nu}{a} \left(1 - e^{-a(T-t)}\right)$$

# Contract Valuation

The market value of a standard participating contract is :

$$V_L(0) = \mathbb{E}_Q \Big[ e^{-\int_0^T r_s ds} \left[ L_T^g + \delta(\alpha A_T - L_T^g)^+ - (L_T^g - A_T)^+ \right] \mathbb{1}_{\tau \geq T} + e^{-\int_0^\tau r_s ds} L_0 \, e^{r_g \tau} \mathbb{1}_{\tau < T} \Big]$$

This is typically a 2D interest rate/default problem in $(r, \tau)$

## Buy and Hold Strategy

$\theta_S$ is invested in risky securities

$\theta_r$ is invested in the risk-free asset

$$A_0 = \theta_S + \theta_r$$

$$A_T = \theta_S \frac{S_T}{S_0} + \theta_r e^{rT}$$

# Constant Proportion Portfolio Insurance

Floor :

$$dF = F\ r\ dt$$

Cushion (difference between assets value and floor level) :

$$\forall t \in [0, T],\ C_t = A_t - F_t$$

Investment in equity (with multiplier $m$) :

$$\forall t \in [0, T],\ e_t = m\ C_t$$

# Constant Proportion Portfolio Insurance

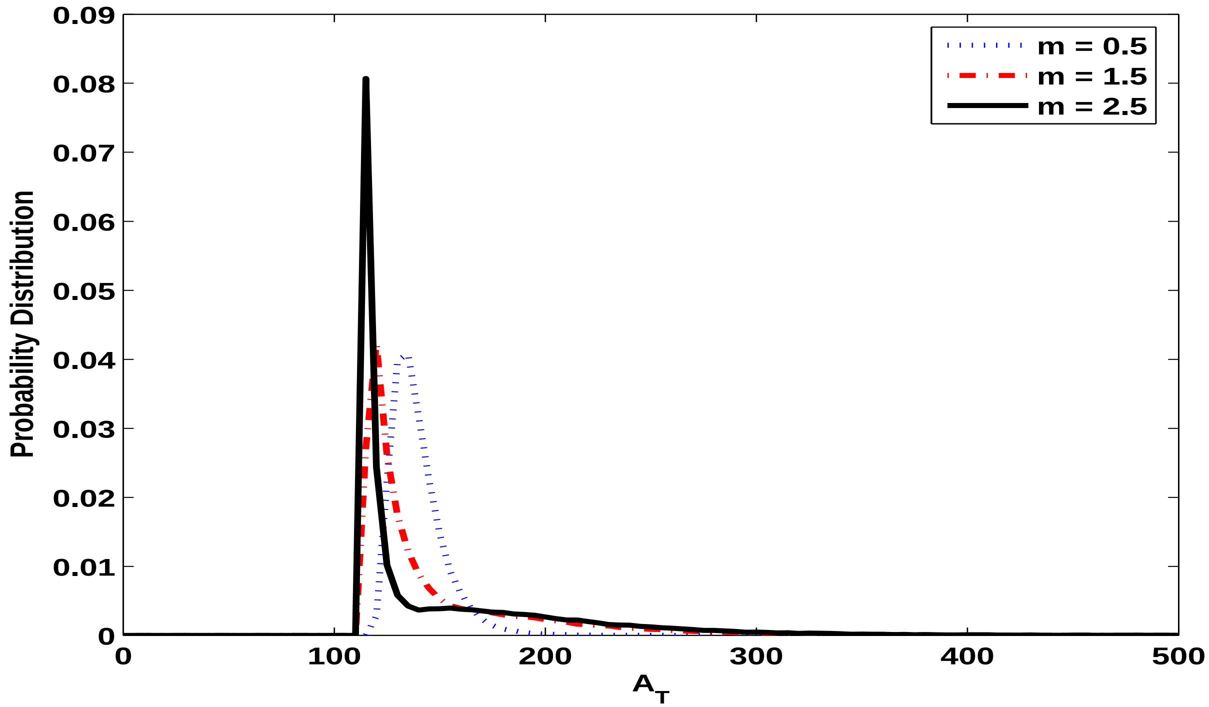
Investment in risk-free asset :

$$\forall t \in [0, T], \; b_t = A_t - e_t$$

To sum up :

$$A_t = F_t + C_t = e_t + b_t$$

# Constant Proportion Portfolio Insurance

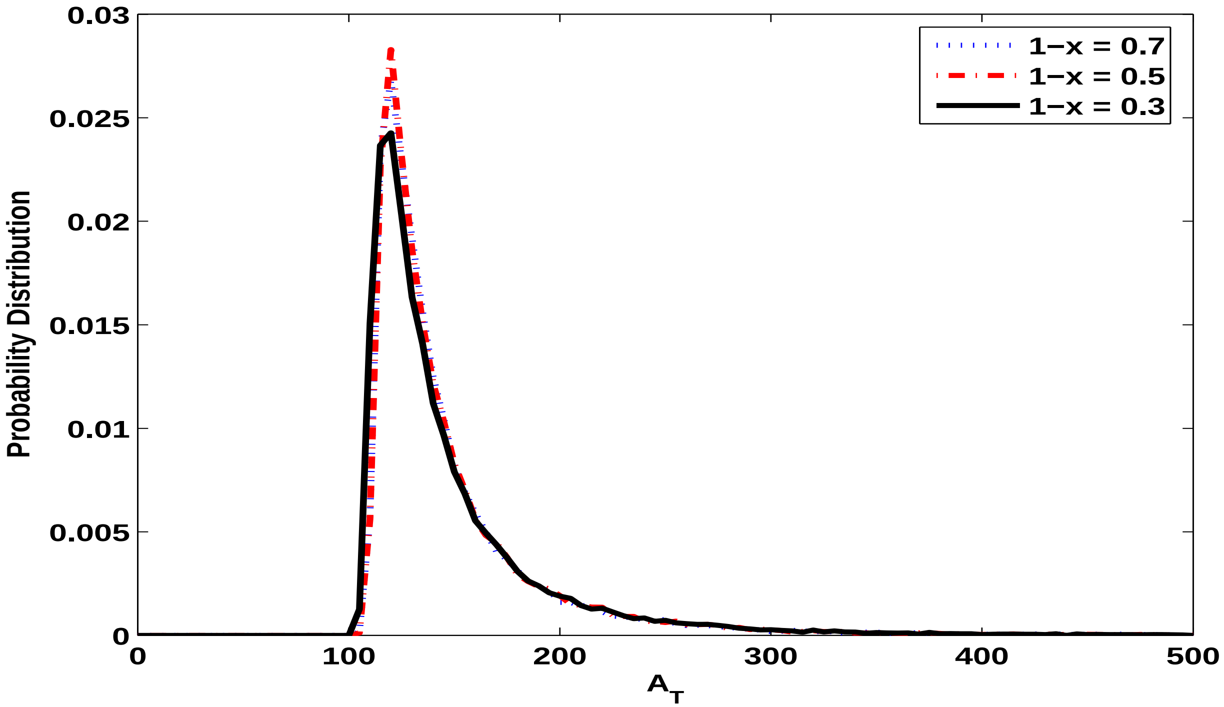


# Equity Default Swaps

An EDS provides protection against a dramatic decline in the price of a risky portfolio, whereas the reference asset of the CDS is a debt instrument, and protection is provided against a possible default.

For example, an equity default swap might provide protection against a 70% decline in the price of the risky portfolio with regard to its initial value. If this event happens, a fixed recovery rate (typically 50%) of the incurred loss, is paid to the investor, and, of course, installments are ceased.

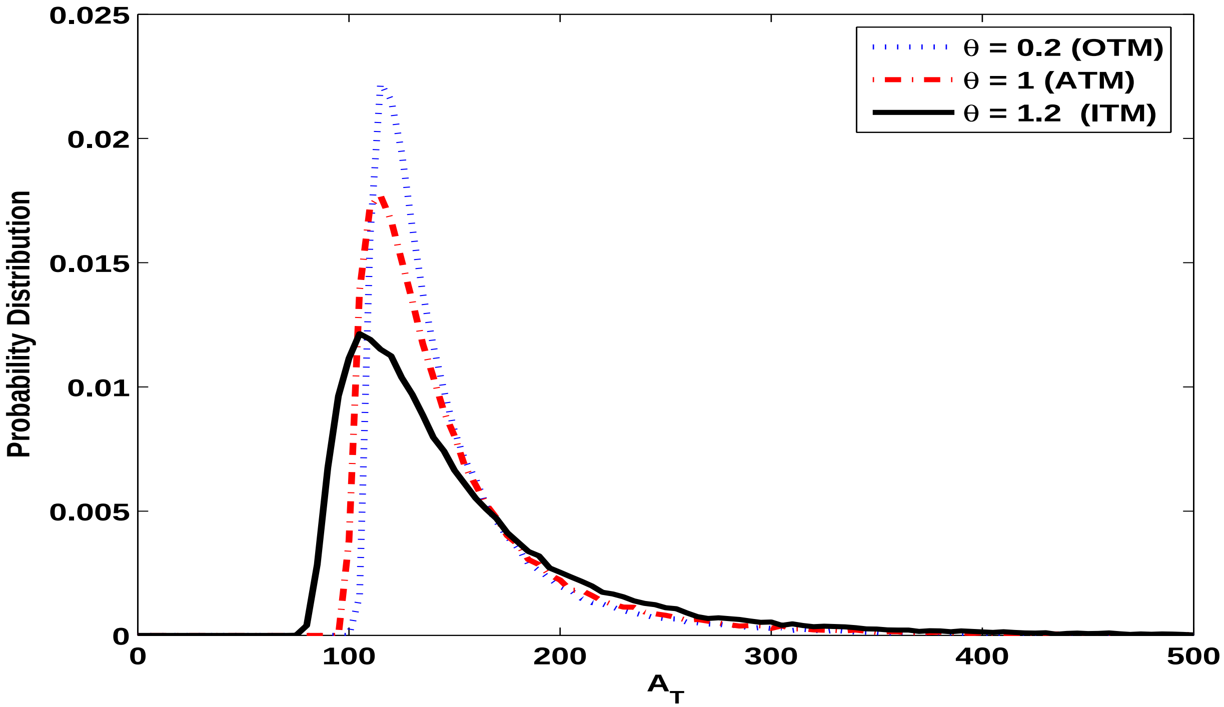
# Equity Default Swaps



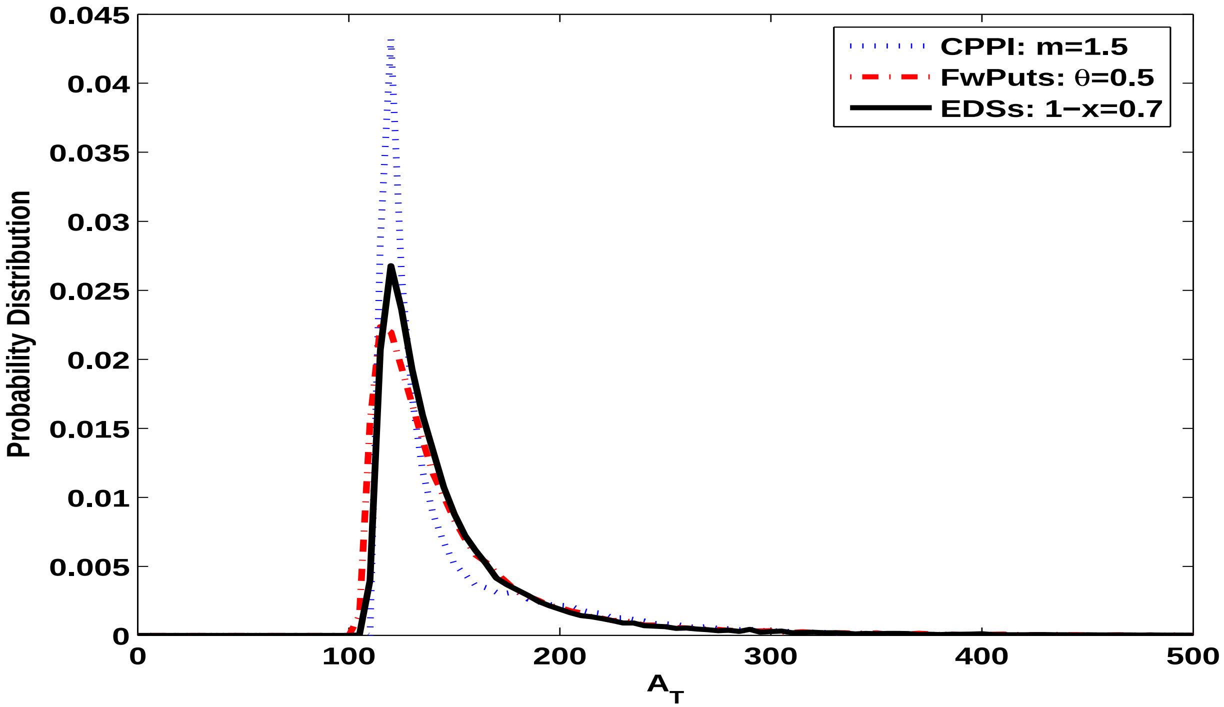
## Forward Start Put Options

To guarantee a fixed rate $g$ over the lifetime $T$ of the contract when there exist no long-term options, it is also possible to buy forward starting put options with shorter maturities, say 1 year. Each option protects the company against a decrease in the assets value for each period. Indeed, a forward starting option is an option that starts at some specified future date $t_0 > 0$ with an expiration $T$ further in the future.

# Forward Start Put Options



# Conclusion



CPPI: m=1.5
FwPuts: θ=0.5
EDSs: 1−x=0.7
Probability Distribution
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
100
200
300
400
500
A_T